Chapter 16

Linear Programming

<u>Linear Inequalities.</u>

All points on a line are represented by an equation. Point to either side of the line are represented by a linear inequality of that equation.

There are 2 steps to identify the linear inequality.

Step 1: **Graph the line**

Step 2: **Use a test point that is not on the line, usually (0, 0).**

If the inequality is true, the arrows point to the test point

If it is false, they point away from the test point.

<u>Example:</u>

Graph the inequality $4x + 3y \geq 12$, indicating the correct half-plane.

<u>Answer:</u>

Step 1: Graph the line $4x + 3y = 12$

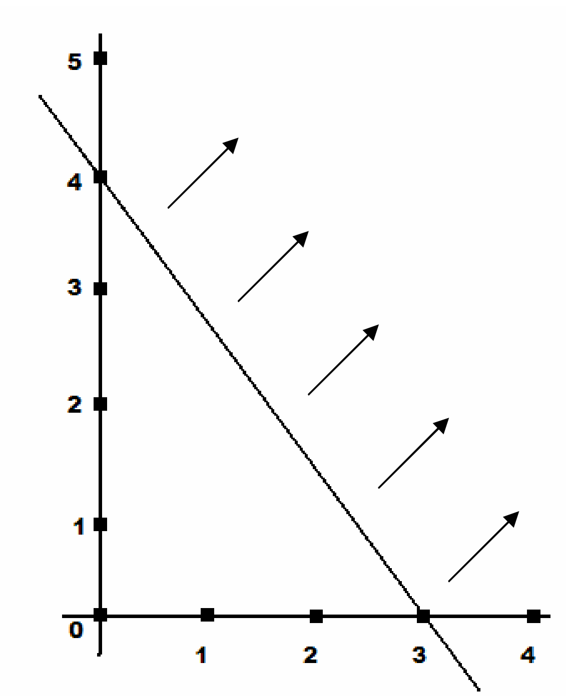
Let $x = 0$	Let $y = 0$
$4(0) + 3y = 12$	$4x + 3(0) = 12$
$3y = 12$	$4x = 12$
$y = 4$	$x = 3$
(0, 4)	(3, 0)

Step 2: Test (0, 0) in $4x + 3y \geq 12$

$4(0) + 3(0) \geq 12$

$0 \geq 12$

FALSE so the arrows point away from (0, 0)



Simultaneous Linear Inequalities.

This is where we are asked to find an area of the graph that satisfy the three inequalities.
To answer this question we follow the same steps as before.
Step 1: Graph the three lines
Step 2: Use a test point to identify what direction they are facing
Step 3: Shade in the common region.

Example:
Illustrate the sets of points (x, y) that simultaneously satisfy the inequality
$x \geq 0$, $x + 2y \leq 4$, $x - y \leq 2$

Answer:
A: $x \geq 0$ is all points on and to the right of $x = 0$

B: $x + 2y \leq 4$
$x + 2y = 4$

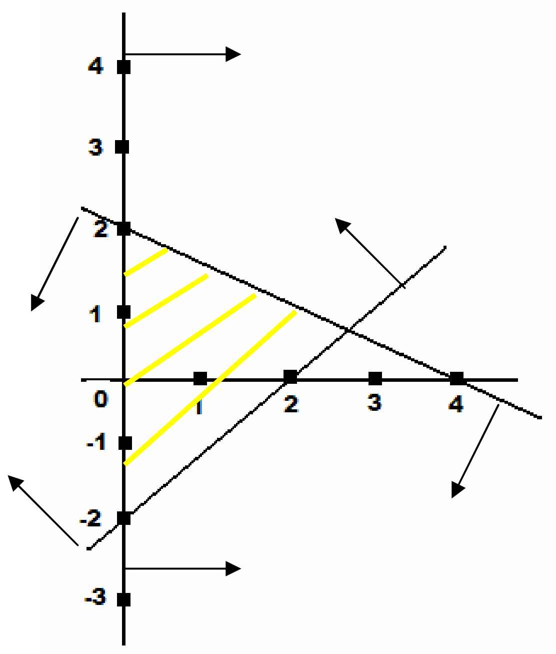
Let $x = 0$	Let y = 0
$0 + 2y = 4$	$x + 2(0) = 4$
$y = 2$	$x = 4$
point = (0, 2)	point = (4, 0)

Test (0, 0)
$0 + 2(0) \leq 4$
$0 \leq 4$ TRUE so arrows point to (0, 0)

C: $x - y \leq 2$
$x - y = 2$

Let x = 0	Let y = 0
$0 - y = 2$	$x - 0 = 2$
$y = -2$	$x = 2$
point = (0, -2)	point = (2, 0)

Test (0, 0)
$0 - 0 \leq 2$
$0 \leq 2$ TRUE so arrows point to (0, 0)



Area shaded in yellow represents the area that satisify the 3 inequalities.

Finding the Inequality.

Example:

The equation of the line K is x – 2y + 2 = 0
The equation of the line L is 3x + y – 6 = 0
Write down three inequalities which define the triangular region indicated in the diagram.

Answer:

The first inequality is $x \geq 0$.

Step 1: Given K : $x - 2y + 2 = 0$

Step 2: We'll assume (0, 3) is in the half plane.

Step 3: Test (0, 3) in $x - 2y + 2 \geq 0$

$$(0) - 2(3) + 2 \geq 0$$

$$-6 + 2 \geq 0$$

$$-4 \geq 0$$

FALSE

Step 4: The statement is false so we reverse the sign of the inequality and we get $x - 2y + 2 \leq 0$

Step 1: Given L : $3x + y - 6 = 0$

Step 2: (0, 3) is in the half plane

Step 3: Test (0, 3) in $3x + y - 6 \geq 0$

$$3(0) + 3 - 6 \geq 0$$

$$-3 \geq 0$$

FALSE

Step 4: This statement is also false so we also reverse the sign of this inequality and get $3x + y - 6 \leq 0$

Maximising and Minimising.

Example:

Illustrate the set K of points (x, y) that simultaneously satisfy the four inequalities.

$x \geq 0$ y ≥ 0 $x + y \leq 6$ $2x + y \leq 10$

Answer:

Step 1: A: $x \geq 0$ is the set of points on and to the right of the y-axis

B: $y \geq 0$ is the set of points on and above the x-axis.

C: $x + y \leq 6$
Line= $x + y = 6$

Let $x = 0$	Let $y = 0$
$0 + y = 6$	$x + 0 = 6$
$y = 6$	$x = 6$
(0, 6)	(6, 0)

Test (0, 0)
$x + y \leq 6$
$0 + 0 \leq 6$
$0 \leq 6$ TRUE

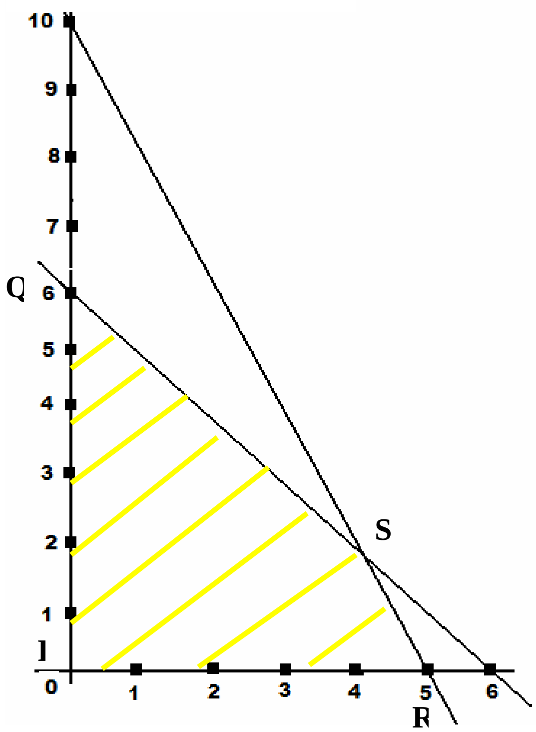
D: $2x + y \leq 10$
Line: $2x + y = 10$

Let $x = 0$	Let $y = 0$
$2(0) + y = 10$	$2x + 0 = 10$
$y = 10$	$2x = 10$ so.. $x = 5$
(0, 10)	(5, 0)

Test (0,0)
$2x + y \leq 10$
$2(0) + 0 \leq 10$
$0 \leq 10$ TRUE

The shaded area K satisfy the 4 inequalities.

Step 2:



Step 3: Let the coordinates of K be p, q, r, s.
From the diagram
p = (0, 0) r = (5, 0)
q = (0, 6) s = (? , ?)

Use simultaneous equations to find S.

$x + y = 6$ (× by -1)
$2x + y = 10$

$-x - y = -6$
$2x + y = 10$
$x = 4$

$x + y = 6$
$4 + y = 6$
$y = 2$
s =(4, 2)